

## What students need to know for...

### AQR

Students expecting to take Advanced Qualitative Reasoning should demonstrate the ability to:

#### General:

- keep an organized notebook
- take good notes
- complete homework every night
- be active learners
- ask questions and participate in class
- seek help outside of class if needed
- work with others
- work with and without a calculator

#### Specific Math Skills

##### 1) Tools of Algebra

- define and use basic concepts and properties of real numbers, operations, and equalities.
- solve and graph one-variable equations with/without absolute value.

##### 2) Linear Relationships and Functions

- define and specify relations and functions by verbal descriptions, lists, and tables
- determine equations for specific functions and relations
- recognize slope as rate of change of one variable in terms of another
- write and interpret a direct variation.

##### 3) Quadratic Equalities and Functions

- write quadratic functions in standard form
- graph quadratic functions to get parabolas
- identify axis of symmetry, intercepts and vertices of parabolas
- solve expressions with radicals
- solve quadratic equations by factoring, finding square roots, graphing, completing the square, quadratic formula
- identify and graph complex numbers
- add, subtract, multiply and divide complex numbers

##### 4) Radical Functions and Rational Exponents

- add, subtract, multiply and divide radical expressions
- rationalize denominators
- simplify expressions with rational exponents
- add, subtract, multiply and divide functions

##### 5) Exponential and Logarithmic Functions

- graph exponential functions
- solve exponential and logarithmic equations

#### **Review Problems:**

\*NOTE: Show all of your work. Your teacher may give a quiz on this material at the beginning of the year. Don't forget to use the reference sheet on page 2. **You should "Google" the topic if you are unsure how to complete the examples. Khanacademy.org has some good instructional videos.**

**SKILL 1: Tools of Algebra**

Solve each formula for the indicated variable:

a)  $A = \frac{1}{2}bh$ , for  $h$

Answer:  $A = \frac{1}{2}bh$                       Original equation

$2 \times A = 2 \times \frac{1}{2}bh$                       Multiply both sides with 2 to clear the fraction

$2A = bh$                                       Simplify

$\frac{2A}{b} = \frac{bh}{b}$                                   Divide both sides by  $b$

$\frac{2A}{b} = h$                                       Simplify

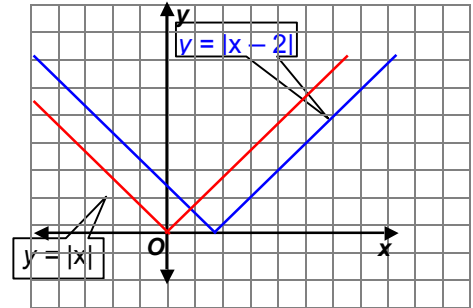
b)  $V = \pi r^2 h$ , for  $h$

c)  $V = s^2 + \frac{1}{2}sh$ , for  $h$

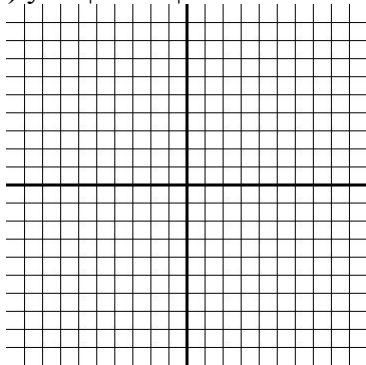
**SKILL 2: Linear Relationships and Functions**

Describe the translation in each case then graph the function.

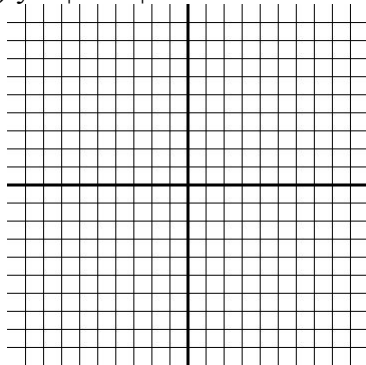
a)  $y = |x - 2|$ . Answer: The graph of  $y = |x - 2|$  is a translation of the graph of  $y = |x|$  moved right 2 units.



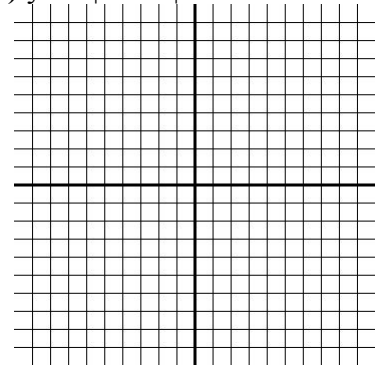
b)  $y = -|x + 6|$



c)  $y = |5 - x| + 3$



d)  $y = 2|x + 3| - 8$



### SKILL 3: Quadratic Equalities and Functions

Solve each equation by factoring.

a)  $5x^3 - 30x^2 = 0$

**Answer:**  $5x^3 - 30x^2 = 5x^2(x) - 5x^2(6) = 0$   
 $= 5x^2(x - 6) = 0$

Factor the GCF.  
Distributive Property

$$5x^2 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = 0 \quad \text{or} \quad x = 6$$

Set each factor equal 0  
Solve each equation

b)  $3x^2 - 6x - 4 + 2x = 0$

c)  $x^2 + 6x + 8 = 0$

d)  $3x^2 = 16x + 12$

### Solve each Equation using Quadratic Formula

a)  $2x^2 - x = 15$

**Answer:**

First, write the equation in the form  $ax^2 + bx + c = 0$  and identify  $a$ ,  $b$ , and  $c$ .

$$ax^2 + bx + c = 0$$

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ 2x^2 - x = 15 \rightarrow 2x^2 - 1x - 15 = 0 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-15)}}{2(2)}$$

Replace  $a$  with 2,  $b$  with  $-1$ , and  $c$  with  $-15$ .

$$x = \frac{1 \pm \sqrt{1 + 120}}{4}$$

Simplify.

$$x = \frac{1 \pm \sqrt{121}}{4}$$

Simplify.

$$x = \frac{1 \pm 11}{4}$$

$$\sqrt{121} = 11$$

$$x = \frac{1 + 11}{4} \quad \text{or} \quad x = \frac{1 - 11}{4}$$

Write as two equations.

$$= 3 \qquad \qquad = -2.5$$

Simplify.

The solutions are  $-2.5$  and  $3$ . Check by substituting each of these values into the original equation.

b)  $x^2 - 4x + 3 = 0$

**Simplify each Complex expression:**

a)  $(-7 + 5i) + (12 + 3i)$

**Answer:**

$$\begin{aligned} &(-7 + 5i) + (12 + 3i) \\ &= (-7 + 12) + (5 + 3)i \\ &= 5 + 8i \end{aligned}$$

Commutative and Associative Properties  
Simplify.

b)  $(2 + 4i) + (4 - 2i)$

c)  $(3 + \sqrt{-4})(4 + \sqrt{-1})$

**SKILL 4: Radical Functions and Rational Exponents**

**Simplify each radical expression:**

a)  $\sqrt{72m^7}$

**Answer:**

$$\begin{aligned} \sqrt{72m^7} &= \sqrt{2 \cdot 6^2 \cdot (m^3)^2 \cdot m} \\ &= \sqrt{2} \cdot \sqrt{6^2} \cdot \sqrt{(m^3)^2} \cdot \sqrt{m} \\ &= 6m^3 \sqrt{2m} \end{aligned}$$

Factor into squares where possible.

Product Property of Radicals

Simplify.

b)  $\sqrt{36} = \underline{\hspace{2cm}}$      $-\sqrt{36} = \underline{\hspace{2cm}}$      $\sqrt{-36} = \underline{\hspace{2cm}}$      $-\sqrt[4]{81} = \underline{\hspace{2cm}}$

c)  $\sqrt{16x^2} = \underline{\hspace{2cm}}$      $\sqrt{x^8y^{18}} = \underline{\hspace{2cm}}$      $\sqrt[3]{x^8y^{12}} = \underline{\hspace{2cm}}$

Solve each equation. Include ALL solutions for x.

a)  $3\sqrt{x} + 3 = 15$

b)  $(x + 5)^{\frac{2}{3}} = 4$

c)  $\sqrt{x + 7} - x = 1$

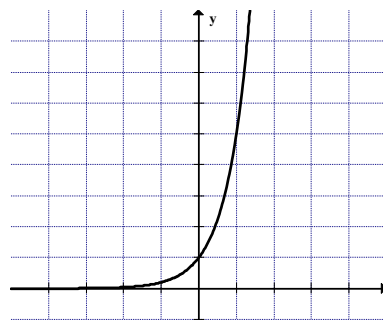
**SKILL 5: Exponential and Logarithmic Functions**

Graph each exponential function. Find the y-intercept, and state the domain and range.

a)  $y = 5^x$ .

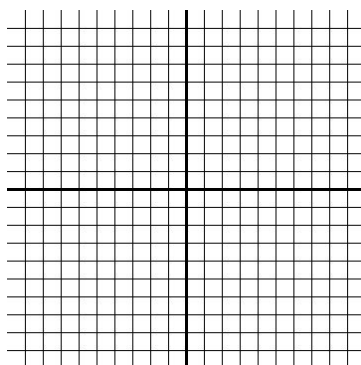
Answer:

$x$	$5^x$	$y$
-2	$5^{-2}$	$\frac{1}{25}$
-1	$5^{-1}$	$\frac{1}{5}$
0	$5^0$	1
1	$5^1$	5
2	$5^2$	25
3	$5^3$	125



Graph the ordered pairs, and connect the points with a smooth curve.  
 The graph crosses the y-axis at 1, so the y-intercept is 1.  
 The domain is all real numbers, and the range is all positive real numbers

b)  $y = \left(\frac{1}{4}\right)^x$ .



### Solve each exponential function

a)  $\left(\frac{1}{2}\right)^{n-1} = 16$

**Answer:**

$\left(\frac{1}{2}\right)^{n-1} = 16$  Original equation

$(2^{-1})^{n-1} = 2^4$  Rewrite  $\frac{1}{2}$  as  $2^{-1}$  and 16 as  $2^4$  so each side as the same base.

$2^{-n+1} = 2^4$  Power of a Power

$-n + 1 = 4$  Property of Equality for Exponential Functions

$-n = 3$  Subtract 1 from each side.

$n = -3$  Divide each side by  $-1$ .

b)  $5^{5n+1} = 125^{n-2}$

### Solve each Logarithmic Equation

a)  $\log_{\frac{1}{8}} a = \frac{2}{3}$

**Answer:**  $\log_{\frac{1}{8}} a = \frac{2}{3}$  Original equation

$a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$  Definition of logarithm

$a = \left[\left(\frac{1}{2}\right)^3\right]^{\frac{2}{3}} \quad \frac{1}{8} = \left(\frac{1}{2}\right)^3$

$a = \left(\frac{1}{2}\right)^2$  or  $\frac{1}{4}$  Power of a Power

b)  $\log_6 (a^2 - 15) = \log_6 (2a)$